NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1274

BEHAVIOR OF FAST MOVING FLOW OF COMPRESSIBLE GAS IN CYLINDRICAL PIPE IN PRESENCE OF COOLING

By G. A. Varshavsky

Translation

"K Voprosu o Povedenii Bystrodvizhushchegosya Potoka Szhimaemogo Gaza v Pryamoi Tsilindricheskoi Trube pri Nalichii Okhlazhdenia." Zhurnal teknicheskoi fiziki (U.S.S.R.). Vol. XVI, no. 4, 1946.

Washington
September 1951

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BEHAVIOR OF FAST MOVING FLOW OF COMPRESSIBLE GAS IN CYLINDRICAL PIPE IN PRESENCE OF COOLING*

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INTRODUCTION

The investigation of the distribution of energy in the flow of a compressible gas that moves without resistance in a straight cylindrical pipe and gives off heat indicates that the existence of a "thermal" Laval nozzle in the supersonic region is possible (reference 1) (that is, $B_a > 1$, the cooling of the flow under these conditions results in a rise in the Bairstow (Mach) number). This interesting result is actually strongly distorted by the effect of the resistance. For the simple case of gas cooling (heat conduction at the wall), the well-known relation between the resistance and the heat transfer makes the existence of a "thermal" Laval nozzle improbable. If only the heat transfer by contact is taken into account, the existence of a "thermal" nozzle is impossible; however, if radiation from the products of combustion is also considered, the "thermal" nozzle is possible only in a narrow range of high temperatures and for large dimensions of the nozzle (pipe diameter).

1. SOLUTION OF EQUATIONS

Assumptions are based on the analysis of the simultaneous solution of the following equations: the momentum equation (in which the friction forces are taken into account by the usual "hydraulic" resistance coefficient)

$$w \frac{dw}{dx} = -\frac{1}{\rho} \frac{dp}{dx} - \zeta \frac{w^2}{2D} \tag{1}$$

the equation of continuity

$$w\rho = \text{constant} \tag{2}$$

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and the heat-transfer equation

$$-Gc_p d\left[T\left(1+\frac{k-1}{2}Ba^2\right)\right] = \alpha\left[T\left(1+\frac{k-1}{2}Ba^2\right)-t_1\right]dS \tag{3}$$

where w (m/sec) is the velocity of the flow at a certain section of the pipe; p (kg/m^2) is the static pressure at the same section; T ($^\circ$K) is the temperature of the gas; ρ ((kg)(sec^2)/m^4) is the density; D (m) is the diameter of the pipe; ζ is the coefficient of resistance; G (kg/sec) is the weight of the gas per second; c_p (cal/(kg)($^\circ$C)) is the specific heat of the gas; α (cal/(m^2)(hr)($^\circ$C)) is the heat transfer coefficient; $Ba = w/c$ is the Bairstow number (Mach number); c (m/sec) is the local velocity of sound; t_0 ($^\circ$K) is the temperature of the wall; and dS is an element of the pipe area that corresponds to an element of the length dx.

The solution of this system for the general case was given by the author and M. D. Weisman in 1934 (reference 2) and leads to complicated expressions not capable of a clear qualitative analysis. In investigating the problem of the formation of a "thermal" Laval nozzle, the system was reduced by the author to nondimensional variables and was solved under the assumption $t_0 = 0$ (that is, at a wall temperature negligibly small in comparison with the stagnation temperature of the flowing gas).

The conclusions as to the possibility of formation of a "thermal" Laval nozzle for $t_0 = 0$ will naturally be the more favorable in this sense because for certain wall temperatures comparable with the gas temperature, the intensity of cooling will be less than in the case considered by the author.

The transformed system of equations is written as follows:

$$\frac{1}{\theta}\frac{d(\beta\theta)}{d\xi} = -\frac{1}{k\pi^2}\frac{d\pi^2}{d\xi} - \zeta\beta \tag{1'}$$

$$\frac{\beta\pi^2}{\theta} = \text{constant} \tag{2'}$$

$$\frac{d\left\{\theta\left[1+\left(\frac{k-1}{2}\right)\beta\right]\right\}}{\theta\left[1+\left(\frac{k-1}{2}\right)\beta\right]} = 4\ Md\xi \tag{3'}$$

NACA comment: The symbol t_1 in equation (3) appears subsequently as t_0. Brackets and braces in equations (3), (1'), and (3') do not appear in the original version but are considered desirable in the interest of clarity.

where $\theta = T/T_0$ is the nondimensional temperature (T_0 is the initial gas temperature); $\pi = p/p_0$ is the nondimensional pressure (p_0 is the initial pressure); $M = \alpha/wgc_p\rho$ is the Margulis criterion $((g)(m)/sec^2)$ is the gravitational acceleration); $\beta = Ba^2$; and $\xi = x/D$ is the nondimensional length.

By elementary transformations, the system (1'), (2'), and (3') is reduced to the equation

$$\frac{d\beta}{d\xi} = \frac{\left[(\zeta - 4M)\beta - \frac{4M}{k}\right] k\beta \left[1 + \left(\frac{k-1}{2}\right)\beta\right]}{1-\beta} \tag{4}$$

that for constant values of ζ and M may be integrated[1]. By simple transformations, corresponding expressions for θ and π can also be written. The relations (4'), (4''), and (4''') are the result of the integration[2]

$$\xi = \frac{1}{4M} \ln \left[\frac{\beta_0}{\beta} \sqrt[7\zeta - 24M]{\left(\frac{1+\lambda\beta}{1+\lambda\beta_0}\right)^{24M} \left[\frac{k(4M-\zeta)\beta+4M}{k(4M-\zeta)\beta_0+4M}\right]^{7\zeta - 48M}}\right] \tag{4'}$$

$$\theta = \frac{\beta}{\beta_0} \left(\frac{1+\lambda\beta_0}{1+\lambda\beta}\right)^{\frac{7\zeta}{7\zeta - 24M}} \left[\frac{k(4M-\quad)\beta_0+4M}{k(4M-\quad)\beta+4M}\right]^{\frac{7\zeta - 48M}{7\zeta - 24M}} \tag{4''}$$

$$\pi^2 = \left(\frac{1+\lambda\beta_0}{1+\lambda\beta}\right)^{\frac{7\zeta}{7\zeta - 24M}} \left[\frac{k(4M-\zeta)\beta_0+4M}{k(4M-\zeta)\beta+4M}\right]^{\frac{7\zeta - 48M}{7\zeta - 24M}} \tag{4'''}$$

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[1]The constant of the values ζ and M and the existence of the normal relations between them for the case of large values of Bairstow (Mach) number has been confirmed several times in both Russian and other foreign literature (references 3 and 4).

[2]Hereinafter $\lambda = (k-1)/2$.

2. ANALYSIS OF SOLUTION

(a) Radiation neglected. The hydrodynamic theory of heat exchange gives the relation between ζ and M

$$\zeta = 8M \tag{5}$$

By substituting equation (5) in equation (4), the following expression is obtained[3]:

$$\frac{d\beta}{d\xi} = \frac{\zeta\beta(1+\lambda\beta)(1-k\beta)}{2(\beta-1)} \tag{6}$$

A study of equation (6) shows the existence of three regions of variation of β:

1. $\beta > 1$. In this case, β drops along the pipe when approaching $\beta = 1$.

2. $1/k < \beta < 1$. Here β increases when approaching $\beta = 1$.

3. $\beta < 1/k$. In this region β drops and approaches the normal flow of an incompressible gas in the presence of cooling. The behavior of β, θ, and π for particular cases is shown in figures 1, 2, and 3.

(b) Radiation considered. In the case of the presence of triatomic products of combustion (water vapor and carbon dioxide) in the flowing gas, a certain quantity of heat will be given to the walls by radiation. This condition leads to an increase in α and a breakdown of relation (5). Determination of the increase in α that is required so that a "thermal" nozzle may exist in region 1 is made possible by equation (6). It is thus necessary that

$$M > \frac{\zeta}{4\left(1 + \frac{1}{k}\right)} \tag{a}$$

[3]The application of relation (6) facilitates finding the heat stress of the endothermal reaction and makes the existence of a thermal nozzle fundamentally possible.

that is

$$M > \frac{\zeta}{685} \tag{7}$$

An increase in the value of α by 17 percent corresponds to equation (7) as compared with the value given by the hydrodynamic theory of heat interchange.

The computations made for a nozzle with a 200-millimeter diameter and by using the air products of gasoline combustion with an excess coefficient equal to 1 indicate that for $p_0 = 1$ at atmosphere, the coefficient of heat transfer increases by 10 percent because of the radiation. In working with a larger air-fuel ratio or oxygen-fuel ratio, compositions for which α increases by more than 17 percent are possible. This increase in α will however occur over a small part of the pipe. After a certain lowering in the temperature and a corresponding decrease in the radiation, a drop in Ba along the pipe begins (as in the usual case).

3. CONCLUSION

1. The hydrodynamic theory of heat exchange applied to the investigation of the possibility of the formation of a "thermal" Laval nozzle on cooling the gas by heat conduction at the wall leads to a negative result; that is, the formation of such a nozzle is impossible.

2. When radiation is considered for the case of the flow of gasoline combustion products in an air-rich mixture, a certain part of the pipe in the region of high temperatures may work as a thermal nozzle.

After a certain lowering of the gas temperature, however, the pipe will operate normally (with a drop in the Bairstow (Mach) number along the pipe). The effect of an increase in Mach number in this case holds only for: (a) relatively large diameters of the pipe and (b) products of combustion obtained in an air-rich mixture. The practical application of the thermal nozzle even under these conditions is in the author's opinion impossible.

Translated by S. Reiss,
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REFERENCES

1. Vulis, L. A.: On the Transformation of Energy in a Flow; etc. Otchet NII.

2. Varshavsky, G. A., and Weisman, M. D.: Bull. NIVK, 1, 1934.

3. Gukhman, A. A., Varshavsky, G. A., and Others: Jour. Tech. Phys., vol. 4, 1934, p. 10.

4. Jung, Ingwar: Wärmeübergang und Reibungswiderstand bei Gasstromung in Rohren bei hohen Geschwindigkeiten. Forschungsheft 380, erganzung 3u Forschung auf dem Gebiete des Ingenieurwesens, Bd. 7, Ausg. B, Sept./okt. 1936.

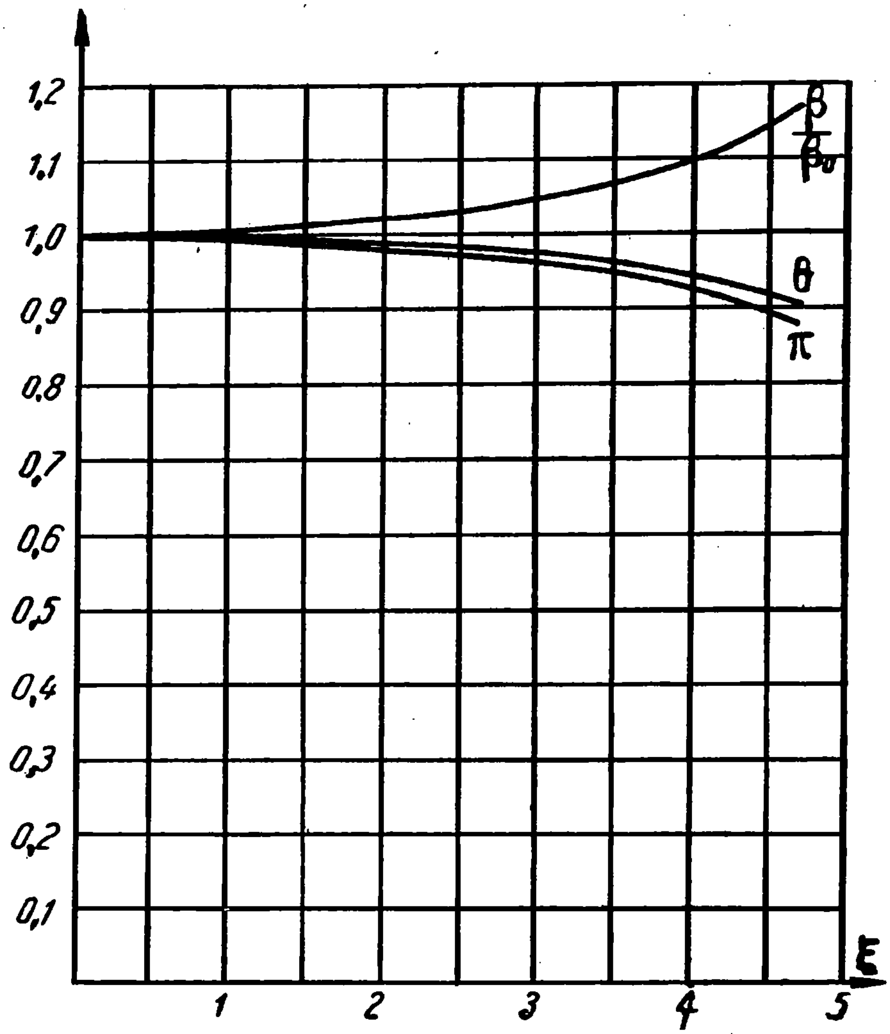

Figure 2. - Dependence of $\frac{\beta}{\beta_0}$, π, and θ on ξ for $\zeta = 0.016$ and $\beta_0 = \frac{6}{7}$.

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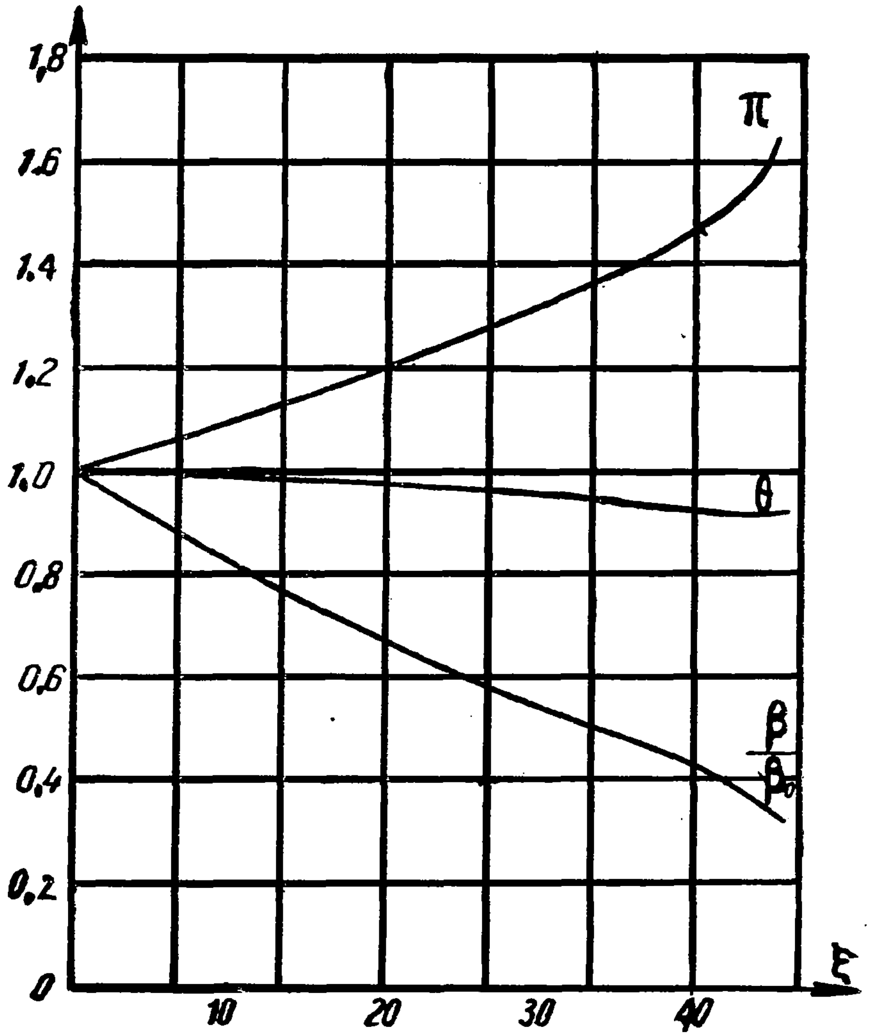

Figure 1. - Dependence of $\frac{\beta}{\beta_0}$, π, and θ on ξ for $\zeta = 0.016$ and $\beta_0 = 3$.